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NUMERICAL SIMULATION OF EXPLOSIVE VOLCANIC ERUPTION IN THE INITIAL STAGE

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In an approximation of the two-dimensional dynamics of a monodisperse collisionless gas suspension, the initial stage of explosive volcanic eruption is modeled. Plots of the amplitude of a shock wave versus the distance in the initial stage are constructed. The general picture of change in the dust concentration near a volcano crater is given. The dependence of the height of a gas-dust column on the explosion energy and the dynamics of ash ejection on volcanic eruption are investigated.

Explosions of volcanoes of the type of Tambora (1815; the energy of eruption in terms of the TNT equivalent is about 25 Gtons) and Krakatau (1833; the energy of eruption is about 300 Mtons) are rather rare events and occur, as a rule, once every several decades or centuries; however their manifestations are so catastrophic and their influence on the environment is so great that evaluation of their consequences is an extremely significant and urgent problem.

It is known [1, 2] that volcanic eruptions usually begin with small ejections of gas and lava, but at the next instants the process can pass to a gas-explosive stage in which a great amount of dust, volcanic ash, and a gas consisting of water vapor and different compounds of sulfur, nitrogen, carbon, and chlorine is ejected into the atmosphere. In reality, volcanic gases are liberated not only during the intense eruption of a volcano, but within an entire period of volcanic activity that sometimes lasts centuries [3].

Ejections of the great amount of dust, gas, and ash on eruption of volcanoes can lead to catastrophic impacts on the biosphere, ecology, and climate of the earth, which is historically evidenced [4]. Quantitative estimates of the hypothesis about the influence of volcanic eruptions on changes in the earth's climate were analyzed for the first time by W. Humphreys [5], who evaluated the possibility of the influence of volcanic eruptions on the climate over the globe. In [6], where the consequences of high-power volcanic eruptions are considered, Budyko points to the probability of climatic fluctuations on great ejections of aerosols. In the survey [7], Toon and Pollack confirm that great volcanic eruptions can exert an influence on climate by means of gas ejection that subsequently leads to the formation of sulfuric acid droplets in the stratosphere, scattering solar radiation, which on marked absorption of the infrared radiation can cause heating of the stratosphere and thus disturb the thermal radiation emitted by the earth's surface [8]. All model calculations of the influence of volcanic aerosols [9] forecast change in the temperature of the earth's surface, while indirect observations indicate that after high-power volcanic eruptions cold weather has been observed.

Thus, high-power volcanic eruptions can entail global climatic changes over the planet.

Moreover, volcanic activity is an important link in maintaining the process of heat and mass exchange between the earth and the atmosphere surrounding it [10].

Recently, in a number of investigations, attempts have been undertaken to model mathematically the phenomena that accompany volcanic activity on the earth. In this connection, we would like to mention the work [11], in which the method of prediction of the propagation of volcanic aerosol ejections in the atmosphere is described, and a series of the works of Barmin and Mel'nik [12–15] concerned with simulation of volcanic eruptions of strong-viscosity gas-saturated magmas. An effort to numerically simulate of gasdynamic processes that take place in the atmosphere after high-power eruption of volcanoes has been made for the first time in [16], where it is suggested that at the initial moment of time the gas and dust are concentrated in the hemisphere and subsequently there occurs expansion of the gas mass heated to 1700 K at its excess pressure of 420 atm. This corresponds to the volcanic eruption of Krakatau

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with an energy of $E = 370$ Mtons [17]. In [18], a model of volcanic explosion is suggested in which the propagation of an explosion-induced shock wave and motion of a dust cloud are considered.

In the present work, mathematical simulation of the behavior of gas and dust in the atmosphere in the initial stage of volcanic eruption is carried out.

Physicomathematical Model. A high-power explosion of a volcano can be subdivided into two stages: 1) a short pulse of an explosion generating a shock wave and related to destruction of the volcano's dome; 2) outflow from the volcanic neck of the basic mass of hot moisture with a temperature close to the temperature of magma consisting of a gas with liquid or plastic fragments. The first stage, which is the subject matter of the present investigation, i.e., the processes directly related to the break-up of discontinuity, usually lasts several tens of seconds. The second stage lasts, as a rule, several days or more. The flow of a gas suspension depends here on the characteristics and velocity of magma, which in the upper part of a volcanic neck passes into the gas suspension. Precisely in this stage of eruption the basic mass of the gas suspension is ejected into the atmosphere; the overwhelming portion of the eruption energy also occurs at this stage.

In the magmatic channel, the gas velocity amounts to 100 m/sec and the density to 10–100 kg/m^3, while the particle size of ash is less than 1 mm [19]. Such particles are entrained, as shown in [19, 20], by a mixture of the gases escaping into the atmosphere.

Let us assume that the products of a volcanic explosion represent a gas suspension of monodisperse spherical particles of dust possessing heat capacity typical of the solid products of volcanic eruptions. As the calculations have shown, variation of the particle size from 0.3 to 1.5 mm leads to a less than 5% change in the characteristics of the gas suspension, which is within the limits of the calculation error and therefore hereafter the dust is considered to be monodisperse.

We consider a volcanic explosion in an approximation of instantaneous destruction of a diaphragm separating the gas suspension heated to the temperature of magma and being in a volcanic neck from the undisturbed atmosphere.

The temperature of the gas suspension is assumed to be equal to the initial temperature of magma, i.e., $\cong$1400 K [19]. We consider not only flow of the gas suspension in the atmosphere but also the change in its parameters in the "idealized" volcanic neck, whose section is assumed to be cylindrical, at different calculated depths. Thus we solve, in fact, the problem on break-up of discontinuity. As the calculation shows, the break-up of discontinuity generates a pressure shock wave propagating in the undisturbed atmosphere and a Riemann rarefaction wave propagating deep into a volcanic neck; these facts correspond well to the known theoretical data on the break-up of discontinuity (see, for instance, [21]).

Let us assume that at the initial moment of time the atmosphere is not disturbed, i.e., all its thermodynamic characteristics coincide with its ordinary values (use is made of the data for the averaged standard atmosphere given in the handbook [22]).

Moreover, we assume that the gasdynamic flow of products of a volcanic explosion in the atmosphere possesses axial symmetry and is described by a system of the equations of mechanics of gas suspensions [23]:

$$\frac{\partial \rho_1}{\partial t} + \frac{\partial}{\partial z}(u_1 \rho_1) + \frac{1}{r}\frac{\partial}{\partial r}(r v_1 \rho_1) = 0 \,, \tag{1}$$

$$\frac{\partial \rho_2}{\partial t} + \frac{\partial}{\partial z}(u_2 \rho_2) + \frac{\partial}{\partial t}(r v_2 \rho_2) = 0 \,, \tag{2}$$

$$\frac{\partial (\rho_1 u_1)}{\partial t} + \frac{\partial (\rho_1 u_1^2)}{\partial z} + \frac{1}{r}\frac{\partial (r \rho_1 u_1 v_1)}{\partial r} = -\left(1 - \frac{3}{2}\alpha_2\right)\frac{\partial p}{\partial z} - \left(1 - \frac{3}{2}\alpha_2\right) n\, (f_\mu)_z + \left(1 - \frac{1}{2}\alpha_2\right)\rho_1 g \,, \tag{3}$$

$$\frac{\partial (\rho_1 v_1)}{\partial t} + \frac{\partial (\rho_1 v_1 u_1)}{\partial z} + \frac{1}{r}\frac{\partial (r \rho_1 v_1^2)}{\partial r} = -\left(1 - \frac{3}{2}\alpha_2\right)\frac{1}{r}\frac{\partial (rp)}{\partial r} - \left(1 - \frac{3}{2}\alpha_2\right) n\, (f_\mu)_r \,, \tag{4}$$

$$\frac{\partial (\rho_2 u_2)}{\partial t} + \frac{\partial (\rho_2 u_2^2)}{\partial z} + \frac{1}{r} \frac{\partial (r \rho_2 u_2 v_2)}{\partial r} = -\frac{3}{2} \alpha_2 \frac{\partial p}{\partial z} + \left(1 - \frac{3}{2} \alpha_2\right) n (f_\mu)_z + \rho_2 g + \frac{1}{2} \alpha_2 \rho_1 g , \tag{5}$$

$$\frac{\partial (\rho_2 v_2)}{\partial t} + \frac{\partial (\rho_2 v_2 u_2)}{\partial z} + \frac{1}{r} \frac{\partial (r \rho_2 v_2^2)}{\partial r} = -\frac{3}{2} \alpha_2 \frac{1}{r} \frac{\partial (rp)}{\partial r} + \left(1 - \frac{3}{2} \alpha_2\right) n (f_\mu)_r , \tag{6}$$

$$\frac{\partial (\rho_1 e_1)}{\partial t} + \frac{\partial (\rho_1 u_1 e_1)}{\partial z} + \frac{1}{r} \frac{\partial (r \rho_1 e_1 v_1)}{\partial r} - \frac{\alpha_1 p}{\rho_1^0} \left[\frac{\partial \rho_1^0}{\partial t} + \frac{\partial (\rho_1^0 u_1)}{\partial z} + \frac{1}{r} \frac{\partial (r \rho_1^0 v_1)}{\partial r}\right] =$$

$$= \left[\left(1 - \frac{3}{2} \alpha_2\right) n (f_\mu)_z + \frac{1}{2} \alpha_2 \left(\rho_1 g - \frac{\partial p}{\partial z}\right)\right] (w_{12})_z + \left[\left(1 - \frac{3}{2} \alpha_2\right) n (f_\mu)_r - \frac{1}{2} \alpha_2 \frac{1}{r} \frac{\partial (rp)}{\partial r}\right] (w_{12})_r + n q_{\Sigma 1} , \tag{7}$$

$$\frac{\partial (\rho_2 e_2)}{\partial t} + \frac{\partial (\rho_2 u_2 e_2)}{\partial z} + \frac{1}{r} \frac{\partial (r \rho_2 e_2 v_2)}{\partial r} = n q_{\Sigma 2} , \tag{8}$$

$$q_{\Sigma 1} = 2\pi a \,\mathrm{Nu}_1\, \lambda_1 (T_2 - T_1) , \quad q_{\Sigma 2} = 2\pi a \,\mathrm{Nu}_2\, \lambda_2 (T_1 - T_2) , \quad \alpha_1 + \alpha_2 = 1 , \tag{9}$$

$$\frac{n (f_\mu)_z}{\alpha_2} = \frac{9\mu_1}{2a^2} (w_{12})_z , \quad \frac{n (f_\mu)_r}{\alpha_2} = \frac{9}{2} \frac{\mu_1}{a^2} (w_{12})_r . \tag{10}$$

We supplement the system (1)–(10) by the equations of state in the form

$$e_1 = \frac{p}{(m-1) \rho_1^0} , \quad e_2 = \int_{T_0}^{T} C_2 (T) \, dT , \quad p = A \rho_1^0 T_1 . \tag{11}$$

We also assume that at the initial moment of time the dust and compressed heated volcanic gas are uniformly distributed in a cylindrical volcanic neck; the atmosphere is not disturbed. On the earth's surface and on the neck walls the zero leakage conditions are set. We consider that at the instant of time $t = 0$ the diaphragm separating the neck and atmosphere is instantaneously destroyed. Then the boundary and initial conditions are written as follows:

$$u_{1,2} (r, z, t = 0) = v_{1,2} (r, z, t = 0) = 0 , \tag{12}$$

$$u_{1,2} (r > r_0, z = 0, t) = v_{1,2} (r = 0, z, t) = 0 , \tag{13}$$

$$v_{1,2} (r = r_0, z < 0, t) = 0 , \quad u_{1,2} (r < r_0, z = -D, t) = u_0 , \tag{14}$$

$$\rho_1 (r < r_0, -D < z < 0, t = 0) = \rho_1^0 , \quad \rho_1 (r, z > 0, t = 0) = \rho_0 (r, z) , \tag{15}$$

$$\alpha_2 (r < r_0, -D < z < 0, t = 0) = \alpha_2^0 , \quad \alpha_2 (r, z > 0, t = 0) = 0 , \tag{16}$$

$$T_1 (r, z > 0, t = 0) = T_0 (r, z) , \quad p (r, z > 0, t = 0) = p_0 (r, z) , \tag{17}$$

$$T_{1,2}\,(r < r_0, -D < z < 0, t = 0) = T_{1,2}\,(r < r_0, z = -D, t) = T_1^0\,, \tag{18}$$

$$\frac{\partial \rho_{1,2}}{\partial r} = \frac{\partial T_{1,2}}{\partial r} = \frac{\partial u_{1,2}}{\partial r} = 0 \quad \text{at} \quad r = 0\,. \tag{19}$$

In the calculations, u_0 was varied. At the boundaries of the calculated region $z = H$ and $r = R$, the gas parameters were considered to be equal to the parameters of the undisturbed atmosphere.

In the system of equations (1)–(12) we introduce dimensionless variables in the following form:

$$r' = \frac{r}{r_0}\,,\ \rho'_{1,2} = \frac{\rho_{1,2}}{\rho_0}\,,\ z' = \frac{z}{r_0}\,,\ p' = \frac{P}{\rho_0 c_0^2}\,,\ T'_{1,2} = \frac{T_{1,2}}{T_0}\,,\ g' = \frac{g r_0}{c_0^2}\,,\ u'_{1,2} = \frac{u_{1,2}}{c_0}\,,\ v'_{1,2} = \frac{v_{1,2}}{c_0}\,,\ e' = \frac{e}{c_0^2}\,; \tag{20}$$

$$r_0 = 300 \ \text{m}\,,\ \ \rho_0 = 1 \ \text{kg/m}^3\,,\ \ c_0 = 300 \ \text{m/sec}\,,\ \ T_0 = 300 \ \text{K}\,.$$

To solve numerically the system of equations obtained in this case, we employed the coarse-particle method developed by O. M. Belotserkovskii and Yu. M. Davydov [24] for solving problems of gas dynamics.

Results and Discussion. The numerical method of solving the problem (1)–(19) written in dimensionless variables in a cylindrical coordinate system has been realized in the form of a program complex that allows modeling of the motion of gas and dust in the earth's atmosphere after explosive eruption of a volcano. The interface of the program complex makes it possible to change the initial conditions of the program, extent of the calculated region, and steps along the space and time coordinates.

In solving the problem numerically, we used the following values of the parameters: $T_1^0 = 1400$ K, $\rho_1^0 = 50$ kg/m^3, $c_0 = 300$ m/sec, and $g = 9.8$ m/sec^2. In the initial stage, a computational grid was adopted having 200 cells over the radius and 200 cells over the height, which allowed consideration of the cylindrical region near a volcano of 30 km height and radius. It was assumed that the depth of the considered part of a volcanic neck was 6 km (40 cells of the computational grid). The calculations have shown that this value does not exert a pronounced influence on the dynamics of the processes initiated by the break-up of discontinuity.

In the calculations, we considered the flow of a gas suspension at different velocities of its entry into a volcanic neck.

Calculations of the velocity and pressure distributions indicate that already at the first minutes after the explosive eruption of a volcano the gasdynamic flow in the atmosphere spans a height of about 20 km. The form of the surface of a propagating pressure shock wave approaches a hemispherical one. Its characteristic form is depicted in Fig. 1.

The dynamics of the wave propagation over the earth's surface in relation to the distance from the place of volcanic eruption at different instants of time and envelopes of the wave amplitudes are shown in Fig. 2 (the initial volume concentrations of dust were assumed to be $\alpha_1 = 0.1$ and $\alpha_2 = 0.01$). The amplitude of the wave front decreases with time and the envelope of amplitudes can be expressed by the relation

$$p\,(r, h) = a\,(h, E, u_0, \alpha_2^0)\, r^{-43} + b\,(h) \tag{21}$$

(E is the effective energy of a volcanic explosion). The amplitude of a shock wave must decrease to zero at $r \to \infty$ so that $b(h) = p_0(h)$ is the background atmospheric pressure at height h. As for the parameter a, at a small height from the epicenter of volcanic eruption it slightly decreases with height. Thus, for instance, for the height $h = 0$ (at a depth of the volcanic neck of $d = 6$ km and initial gas density of $\rho_1^0 = 50$ kg/m^3), $u_0 = 15$ m/sec, and $\alpha_2 = 0.1$ we obtain $a(h) = 19.7$; for $\alpha_2 = 0.1$ we obtain $a(h) = 14.3$. The relationship between the parameter a and the initial gas density in the volcanic neck at $d = 6$ km and $\alpha_2 = 0.1$ is plotted in Fig. 3.

The dependence of the amplitude of the shock wave at different instants of time on the initial gas density in a volcanic neck ρ_1^0 is shown in Fig. 4.

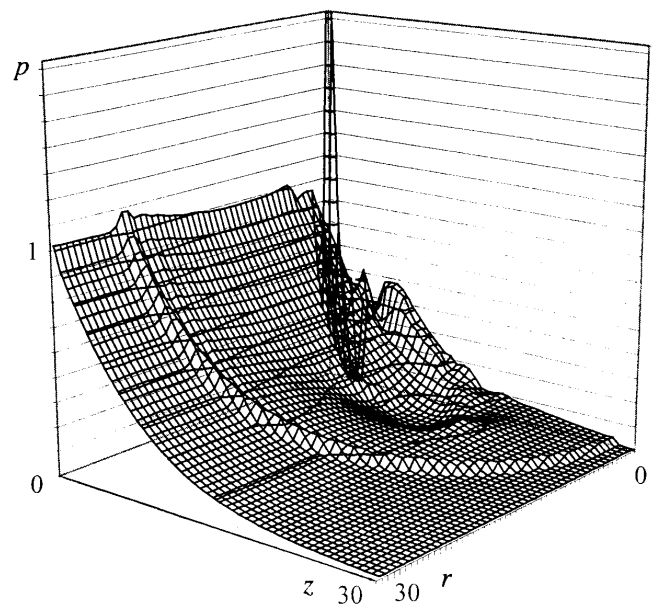
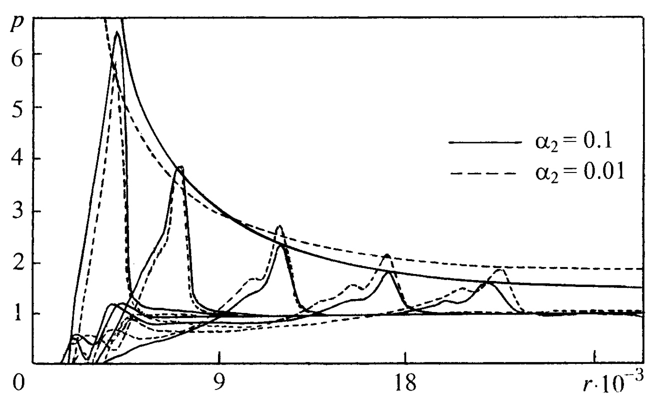


Fig. 1. Characteristic form of a pressure shock wave ($u_0 = 15$ m/sec, $\alpha_2 = 0.1$, $t = 60$ sec). r, km; z, km; p, atm.



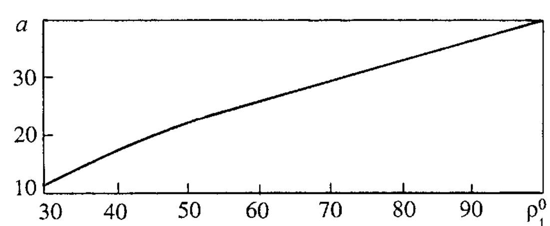


Fig. 2. Amplitudes of the propagating shock wave at different initial concentrations of the dust and envelopes of the amplitudes ($h = 0$, $u_0 = 15$ m/sec, $\alpha_2 = 0.1$, and $\alpha_2 = 0.01$). r, km; p, atm.

Fig. 3. Empirical parameter a versus the initial gas density in a volcanic neck, a, Pa/m$^{4/3}$; ρ_1^0, kg/m^3.

From the foregoing it follows that the amplitude of the shock wave decreases from the axis running through the epicenter of explosive eruption of a volcano (at least up to heights less than 10 km) as $r^{-4/3}$.

Formula (21) allows prediction of the possible destructions by a shock wave formed on explosive volcanic eruption. Shock waves produced by volcanic explosions are the main destroying factor. At an excess pressure of 0.35 atm, they can destroy buildings. If the pressure in a shock wave is hundredth fractions higher than atmospheric pressure, only glasses in windows are broken up.

It is a known fact that on explosion of the Krakatau volcano glasses were knocked out even at a distance of 100 miles [17]. Evaluations made with the use of formula (21) have shown that at this distance the excess pressure is equal to 0.04 atm, which indirectly supports the suggested physicomathematical model.

Results of the numerical calculation within the framework of this model indicate that in the initial stage of eruption gases escape with supersonic velocities. If we employ the initial and boundary conditions given above, we obtain a more than threefold velocity of outflow in the initial stage as compared to the sound velocity.

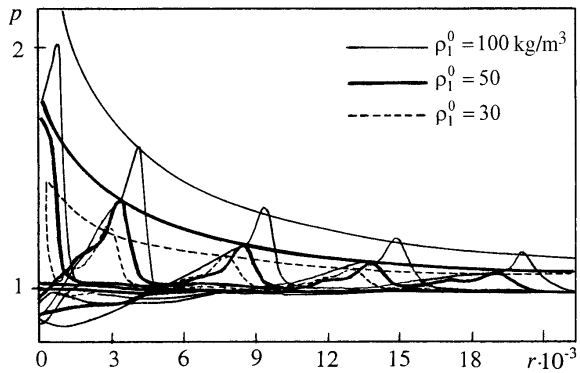


Fig. 4. Dynamics of propagation of the shock wave and envelopes of the amplitudes at different initial densities of the gas in a volcanic neck ρ_1^0. *r*, km; *p*, atm.

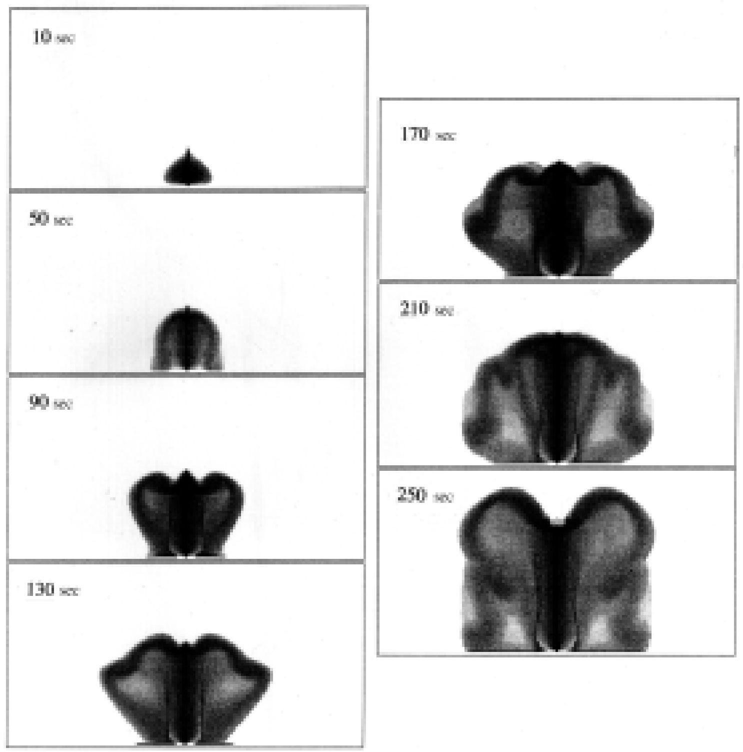


Fig. 5. Evolution of a dust cloud formed on volcanic eruption ($u_0 = 15$ m/sec, $\alpha_2 = 0.1$).

The calculation shows that in the course of evolution of the flow of a gas suspended matter therein vortices are formed.

The process of growth of a dust cloud produced by eruption is illustrated in Fig. 5. Here, the saturation of color is proportional to the logarithm of the volume concentration of the dust. The growth of the central gas-dust column continues several minutes, and afterwards the dust begins to descend. The calculations have shown that more than 90% of the dust ejected into the atmosphere as a result of explosion falls into the stratosphere. For the above values of a volcanic neck and an initial dust concentration of $\alpha_2 = 0.1$, the mass of the dust ejected on explosion is of about $4 \cdot 10^4$ tons. In the absence of wind, at a distance of 4–5 km from the epicenter the dust ejected by explosion forms a 20–25-cm layer on the earth while at a distance of 8–10 km, only a 1–2-cm layer.

A calculation of the vertical distribution of the volume concentration of dust in the atmosphere at the instant of time $t = 60$ sec for different initial gas densities in a volcanic neck has shown that the ash initially uniformly distributed over the neck height forms a distinctive "missile" with a concentration close to the initial one; this ash accumulation moves further as a whole to a maximum height.

After destruction of the volcano's dome and break-up of discontinuity the main factor in the parameters of the gas suspension that determines the height of a gas-dust column is the velocity of inflow of the extremely hot gas suspension from the site of eruption to the volcano channel. From the law of conservation of energy we obtain $H \le u_0^2/2g$, which for the velocities of outflow not exceeding the sound velocity gives a height of the gas-dust column of about several kilometers. This estimate is indicative of the fact that the greater portion of the dust ejected into the stratosphere arrives there in the initial stage of eruption under the action of flows that have emerged as a result of the break-up of discontinuity.

The rate of precipitation of the ejected dust attains about 10 m/sec. The estimations carried out show that if at a height of several kilometers there are constant winds with velocities of about tens of meters per second, the dust can be carried away by the winds for distances up to 100 km and precipitate there to form a layer of more than 1 cm thickness.

If we apply the formula for evaluation of the energy of volcanic eruption with respect to a shock wave that emerges according to the suggested model of the initial stage of eruption, we obtain that at the initial conditions under consideration a volcanic explosion is equivalent to a nuclear explosion with an energy of $E \cong 400$ Mtons of TNT.

NOTATION

ρ_1 and ρ_2, reduced densities of the gas and dust — masses of the corresponding phases per unit volume of the mixture, kg/m^3; r, radial coordinate, m; z, vertical coordinate, m; t, time, sec; u_1 and v_1, vertical and radial velocities of the gas, m/sec; u_2 and v_2, vertical and radial velocities of the dust, m/sec; f_μ, volume force of the interface interaction, N/m^3; n, volume concentration of the dust, 1/m^3; g, free-fall acceleration, m/sec^2; p, gas pressure, Pa; $q_{\Sigma 1}$ and $q_{\Sigma 2}$, heat fluxes across the surface of particles to the gas medium and into particles, respectively; J/m^2; T_1 and T_2, temperatures of the gas and dust, K; λ_1 and λ_2, thermal conductivities of the gas and dust, W/(m·K); e_1 and e_2, internal energy of unit mass of the gas and dust. J/kg; a, particle radius of the dust, m; μ_1, dynamic viscosity coefficient of the gas, kg/(m·sec); $w_{12} = (u_1 - u_2,\ v_1 - v_{2)}$, gas velocity relative to dust particles, m/sec; $C_2(T)$, heat capacity of the dust substance, J/(kg·K); A, gas constant, m^2/(sec^2·K); u_0, velocity of inflow of the gas suspension into a volcanic neck, m/sec; $\rho_0(r, z)$, undisturbed density of the atmosphere, kg/m^3; $p_0(r, z)$, undisturbed pressure of the atmosphere, Pa; $T_0(r, z)$, undisturbed temperature of the atmosphere, K; T_1^0, initial temperature of the gas and dust in the volcanic neck, K; D, depth of the volcanic neck, m; H, height of the calculated region, m; R, radius of the calculated region, m; $a(h, E, u_0, \alpha_2^0)$, parameter of the empirical formula for determination of the pressure shock wave, Pa; α_1 and α_2, volume concentrations of the gas and dust; Nu_1 and Nu_2, Nusselt numbers for the gas and dust; m, adiabatic exponent of the gas; α_2^0, initial volume concentration of the dust in the volcanic neck.

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